# An Electric Analog
# For Computing Direct Surface Runoff

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CONTENTS

# AN ELECTRIC ANALOG FOR COMPUTING DIRECT SURFACE RUNOFF [1/]

By

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## INTRODUCTION

The purpose of this study was to demonstrate the feasibility of an electric analog device for routing runoff to generate hydrographs on experimental watersheds. A small, inexpensive analog model was desired for quickly routing runoff from hourly records of precipitation on headwater areas. Even the most simple routing through multiple stages, as if the watershed and channels act like a series of reservoirs, is adequate if time intervals are very short. The use of many increments of time and storage would complicate any long-hand or graphical computation previously known, but this is no problem with fast electronic devices. A rapid solution could be obtained by utilizing the analogy between the flow of electricity in a passive network of resistors and capacitors as used in oil-reservoir simulations and soil-moisture movement studies.

A simplified, schematic explanation of the hydrology of watersheds is first outlined to show that they function like a series of reservoirs. After derivation of the analogy to a reservoir in electrical units, a description of some construction and of calibration details completes the discussion of electronic equipment. Finally, the operation of this analog model is shown to simulate the time-consuming routing computations previously required. Some actual hydrograph computations illustrate the applicability of this model at a cost of about \$100. The term analog "model" is more descriptive of this network for solving the storage function by the use of common radio parts than is the term "computer."

## TERMINOLOGY

| Symbol | | Unit |
|---|---|---|
| | **Electrical** | |
| $I_{1,2,3}$ | = Current (ma = milliamperes) | amperes |
| $L_{1,2,3}$ | = Inductance | Henrys |
| $R_{1,2,3}$ | = Resistance (K = 1,000 ohms) | ohms |
| $V_{1,2,3}$ | = Voltage | volts |
| $C_{1,2,3}$ | = Capacitance ($\mu$f. = microfarads) | farads |
| Qc | = Charge on capacitors | coulombs |
| Ts | = Time of storage | seconds |

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| Symbol | | Unit |
|---|---|---|
| | **Hydraulic** | |
| $I_i$ | = Inflow | c.f.s. |
| $I_o$ | = Outflow | c.f.s. |
| $Q_o$ | = Initial discharge | c.f.s. |
| $Q_t$ | = Discharge at time | c.f.s. |
| t | = Time | seconds |
| e | = 2.7183 | |
| X | = Relative effect of inflow and outflow, when | |
| k | = "Muskingum" routing coefficient | |
| | **Hydrologic** | |
| Q | = Direct storm runoff | inches |
| $Q_{2,3}$ | = Accumulated runoff for periods | inches |
| $\Delta Q_{1,2}$ | = Increment of runoff | inches |
| $\Sigma Q$ | = Mass direct runoff | inches |
| P | = Storm rainfall | inches |
| $P_{2,3}$ | = Accumulated rainfall for periods | inches |
| $P_{1,2}$ | = Rainfall for periods | inches |
| $\Sigma P$ | = Mass precipitation | inches |
| S | = Storage | inches |
| K | = Recession constant | |

## ANALOG DEVICES

Analog devices depend on--and are named for--the use of analogies, which are similarities of properties of relationships. As stated by Rogers and Connolly (13),[3/] simulation results whenever a physical model is represented in such a way that its characteristics, parameters, and behavior can be easily identified, adjusted, and studied as accomplished in the use of analog models. The term "analog computer" refers to operational amplifiers, which are assembled into a computer for general purposes. Such a computer must be capable of (1) multiplying constants, (2) adding variables, (3) multiplying variables, (4) integrating variables with time, or (5) generating a function of a variable.

All analogs could be called computers, but all computers do not operate as analog models. One major type of computer is called digital--which deals in numbers according to an ordered sequence of simple arithmetic operations. It computes by repeatedly refining an approximation so that its accuracy is potentially unlimited. Another type of computer is some sort of physical model that is set in operation to generate a solution. This analog unfortunately is limited in accuracy by the physical elements of the system. The most direct analog "model" (not usually called a computer) performs like a scale model under experimentation in a hydraulic laboratory. It warrants being in a class by itself--the passive network.

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3/ Underscored numbers in parentheses refer to Literature Cited at the end of this report.

As utilized by Harder and others (6), electric analogs are models of a dynamic system where flow networks can be integrated by instantaneous electrical currents. Such electric analog models have been used by the petroleum industry, according to Scheidegger (14), and should have a similar application to ground-water problems. Problems involving ground-water behavior, as studied by Skibitzke and Brown,[4] can be solved by using electric analogs where solutions are unattainable in any other way. There is no limit to the application of such electric analogs to hydrologic problems, according to Tribus (15), since it is possible to construct analogous electrical networks for every dynamic system.

Electric analogs for routing streamflow to generate hydrographs on small watersheds are a recent development. Linsley, Foskett, and Kohler (9) in 1948 first utilized an electronic routing machine to route floods from basins of up to 15,000 square miles. In 1952 Paynter (11) demonstrated the analogy between flood routing by admittance methods and transients in electrical transmission lines. Glover (4) in 1953 presented an electronic circuit for an analog model which provided for the square-law resistance. And in 1956 Rockwood and Hildebrand (12) described an electrical circuit for multiple-stage storage routing on large rivers.

Messerle (10) first reported an electronic high-speed simulator developed at the University of Melbourne in 1953. Japanese developments were described in 1955 and 1957 by Ishihara and Ishihara (7) and Ishihara (8). At about this same time, foreign articles by Dziatlik (3) and Halek (5) mentioned analog apparatus. By 1958 electric analogs were mentioned in the literature of most countries.

## WATERSHED ANALOGY

Basically, the watershed analogy can be represented as an exchange of water between storage and flow or losses. The time pattern of precipitation is a variable input following the trace of a recording gage. The first losses are intercepted by vegetation, which is shown as part of evapotranspiration in figure 1.

The first increment of runoff could be called channel interception or the rain falling directly on the wet surface area of the open channel. Direct evaporation from water surfaces could be extracted here and combined with other losses. Other increments of overland flow follow each other down the channel after separation at the soil surface by infiltration. All increments of runoff to this point are routed through storage: First through detention on the rough land surfaces or humus ($A_0$) layer; then as channel storage, including bank storage, less other riparian losses downstream. These losses and storages are shown diagrammatically in figure 1.

Moisture entering the topsoil ($A_1$) is available for percolation or is eventually extracted by the process of evapotranspiration in proportion to temperature or other controlling factors. As far as immediate direct runoff

4/ Skibitzke, H. E., and Brown, R. H. Analysis of hydrologic systems. U.S. Geol. Survey unpublished paper, 10 pp. 1961.

is concerned, the characteristics of the detention volume in the soil profile determine the percolation rate of free water into the next incremental volume of storage, shown as subsoil ($B_1$). Here again, retention is trapped and released gradually either to direct runoff laterally if there is a limiting layer ($B_2$), or to percolation downward into the rock mantle ($C_1$), as shown in figure 1.

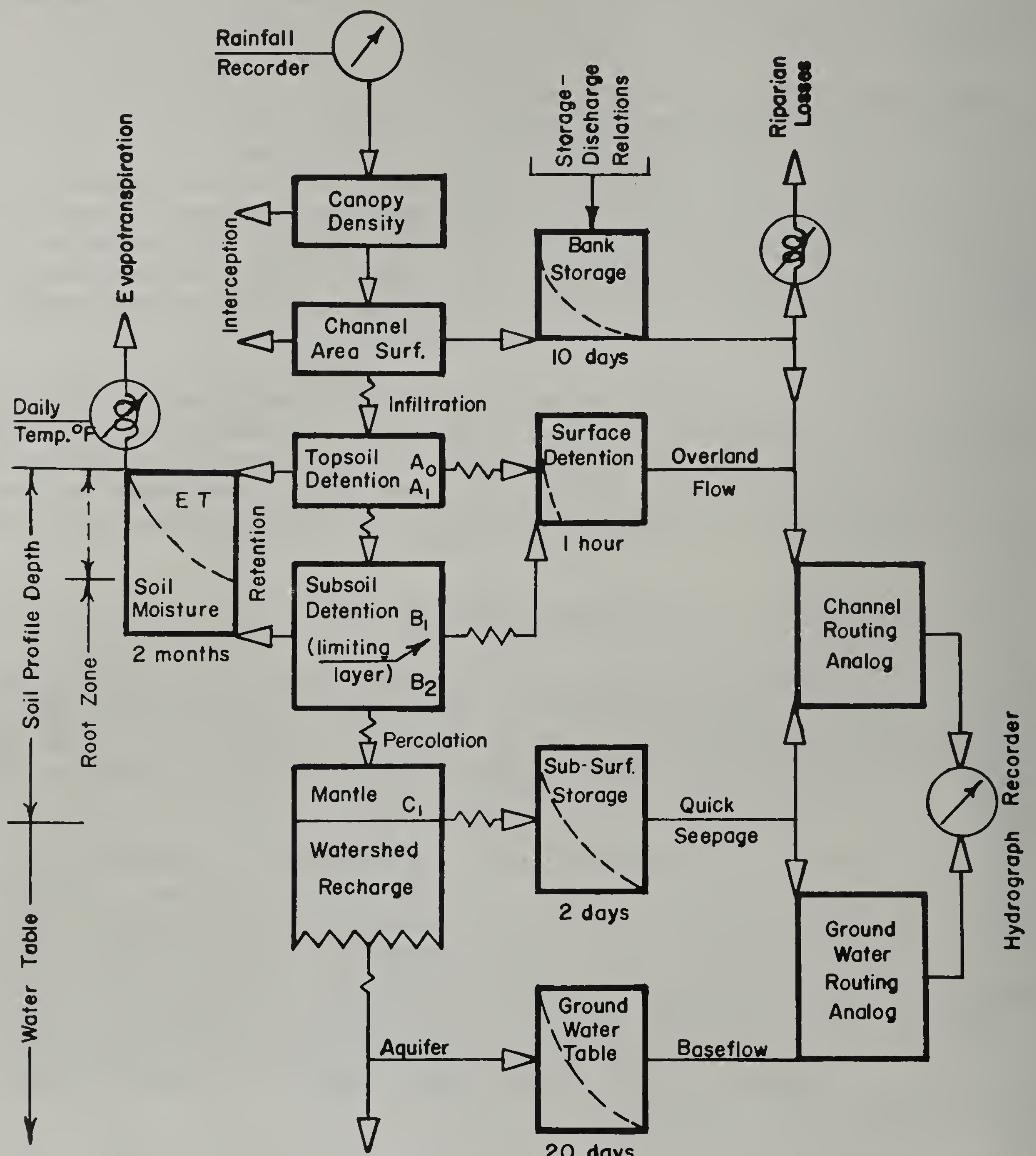



Figure 1.--Schematic diagram of watershed analog.

A shallow profile might contribute quick seepage at this point after saturation, as represented by a limited storage-discharge relation. A deep porous mantle would continue to receive watershed recharge, which drains out much later through an aquifer, as represented by a great volume of ground water storage. Wherever the increments of outflow drain into the channel from each increment of storage, there would be a division of the flow (for routing purposes) between the limited storage in channels relatively far above the point gaged and the greater storage in underground sources leading eventually to the gaging station. The runoff component of overland flow and quick seepage (usually called direct runoff), as shown in figure 1, is quickly routed through a channel analog representing the drainage patterns. Base flow or ground water is similarly routed through a ground water analog capable of a much greater time of storage.

The multiple-stage storage routing method is outlined in figure 2 as it applies to a complex channel system. This represents a watershed that must be divided into 8 to 10 subareas in order to evaluate the runoff from different rainfall patterns and complex soil-cover conditions. Thus, at many points down the main channel either tributary or local inflow must be added.

In order to express variable channel cross sections, slopes, and roughnesses, which could be computed to give different storage volumes, some four or five measured sections are assumed for each reach. Thus, there are perhaps five stages through which to route the water down each reach of the main channel between subareas. To get from one cross section to the next, the flow must be routed through the storage, which (between closely spaced cross sections) can be assumed to act like a series of reservoirs.

Figure 3 has been adapted from Bruce (1) to represent the behavior of underground reservoirs--in this case, the ground-water table. Recharge at any point would be distributed through a series of storage units until discharged at a distant point. Subsurface flow would respond to disturbances of the pressure surface represented by water table levels. Unit recharge on a watershed slope must move, toward an outlet, through a network of pores or minor underground channels of constantly changing diameter. This can be represented simply as a series of tiny tubes, end-to-end, with storage changing between sections.

A watershed could be modeled after figure 3 as a three-dimensional network of resistors and capacitors and stacked to the shape of the water table, since the governing fluid equations are identical in form to those representing the behavior of such an electric circuit. Scheidegger (14) has shown that such analogs will solve complex flow equations and that the proper size of capacitors and resistors may be selected by trial and error to match a known past history of behavior.

Figure 4 shows the analog model for several stages of storage which could represent either the channel reaches for surface runoff, as previously discussed and represented in figure 2, or the units of storage available in the soil or rock mantle for the subsurface or ground water flow shown in figure 3. The schematic diagram in figure 4 is drawn simply as if the hourly rainfall entered through a funnel into a system of pipes and tanks. Each tank

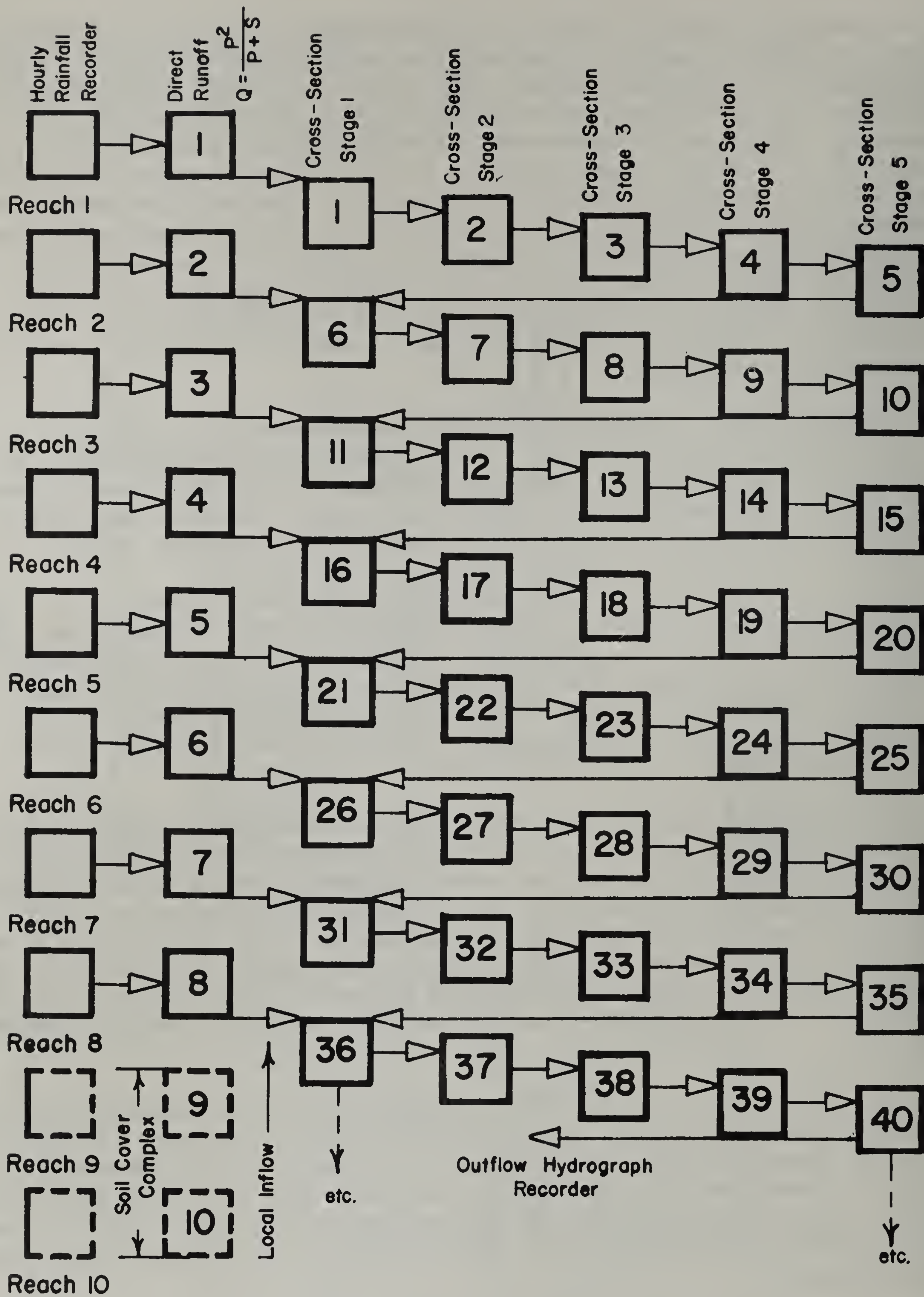



Figure 2.--Multiple-stage storage routing.

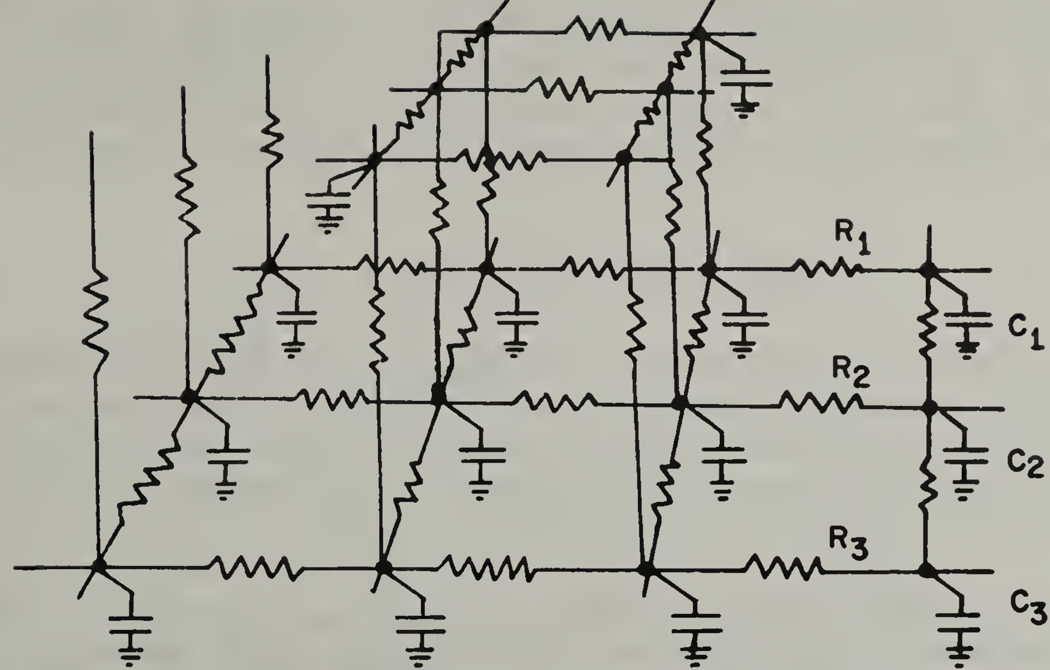



Figure 3.--Part of a three-dimensional electrical representation (1).

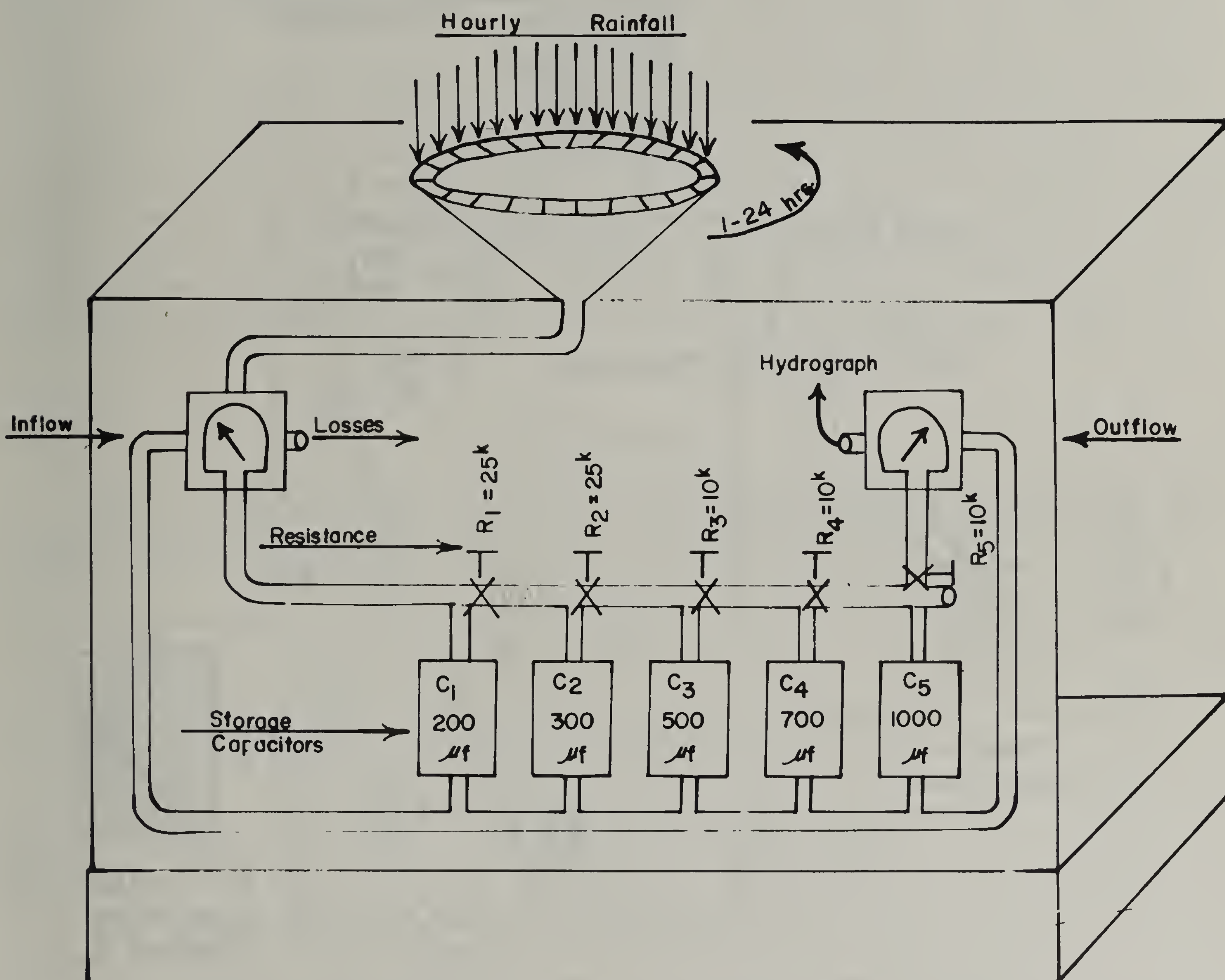



Figure 4.--Sketch of the analog model.

is analogous to storage in charging a capacitor (C), and flow between tanks is controlled by a valve that is analogous to a variable resistor (R). Thus, the time it takes for a tank to empty would be a function of capacity and rate of outflow. The final outflow past the last stage would be a hydrograph integrating the effect of all prior stages of storage and resistance.

## TEST MODEL OF ELECTRIC ANALOG

The analogy between an electric current in a capacitor-resistor circuit and water flowing through storage can be derived for the simple case of a reservoir. If several reservoirs are connected, then the equations become too complex for easy solution. Therefore, a simple model of the network is the answer.

A circuit diagram for an electronic flood-routing device developed by the U. S. Weather Bureau in 1948 and built commercially for each of the River Forecasting Centers is shown in figure 5. Resistances and capacitors,

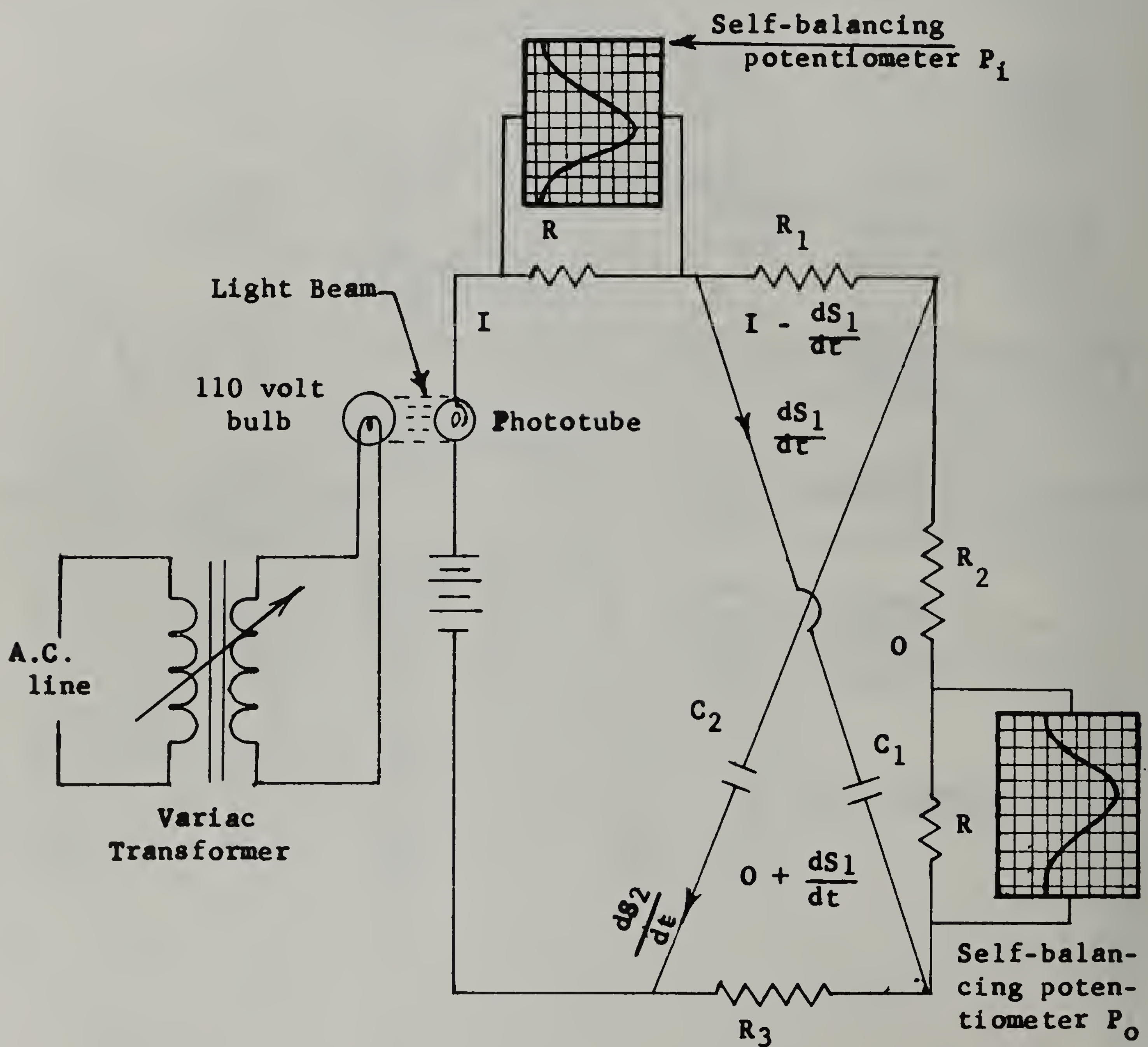



Figure 5.--Circuit diagram for electronic flood-routing machine (9).

cross connected, are analogous to the "Muskingum" method of routing, where S is storage, k is the time of storage, and X is the relative effect of inflow and outflow in the equation:

$$S = k\,[XI + (1-X)O]$$

where,

$$k = 2C_1\ (R_1+R_2) \text{ and } X = \frac{R_1}{2(R_1+R_2)}\ .$$

This flood-routing device represents only one stage so that outflow has to be recorded and fed back into the input in order to duplicate multiple stages of flood routing. When X = 0.0, storage is related to discharge alone as in a reservoir and when X = 0.5, a channel is represented with inflow and outflow equally important in determining storage within a river reach.

Another flood-routing analog, using two stages of resistors and capacitors, was built for the U.S. Corps of Engineers at Portland, Oreg., in 1956. The circuit diagram of this analog, as described by Rockwood and Hildebrand (12), is shown in figure 6. It was a small battery-powered test model, only about 12 inches long, having variable rheostats across each bank of capacitors so that leakage currents could be increased. Thus, an exponential relation of storage and discharge could be approximated by slowly reducing the resistance as outflow increased. Such an operation releases tremendous surges from storage and would be difficult to link mechanically to a variable output. Leakage across the capacitors is analogous to actual losses of input, so that output would have to be adjusted to unity. Limited use was found for this experimental development because flood-routing problems at about that same time were programmed to digital computers made available to U.S. Corps of Engineer offices.

## Electric Analogy

The electric analogy to reservoir-type storage, upon which the multiple-stage storage routing analog is based, has been described by Rockwood and Hildebrand (12). Inflow is related to outflow by the continuity equation

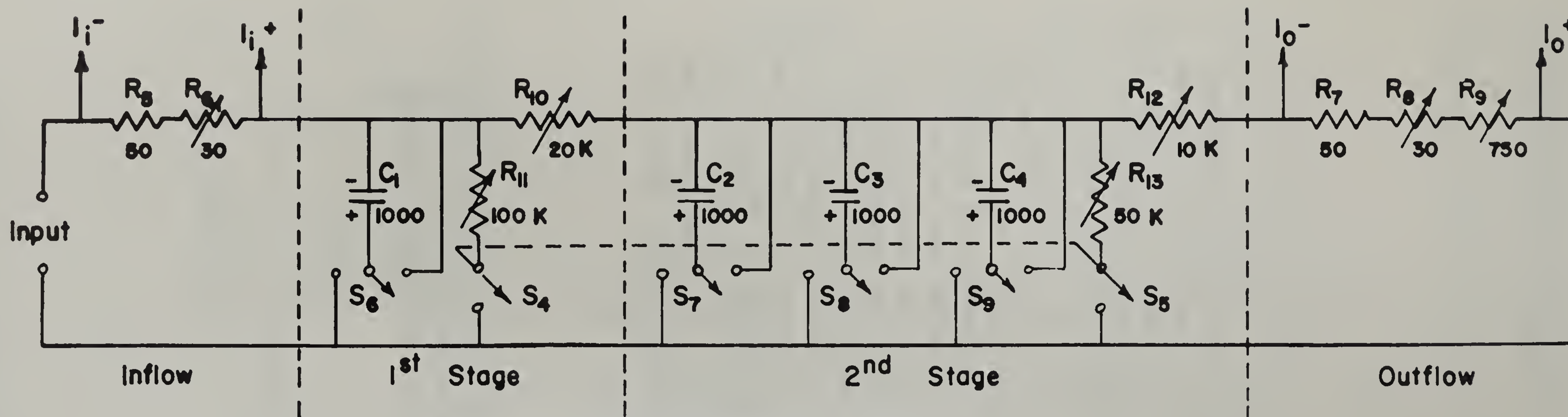
$$I = O + \frac{dS}{dt} \tag{1}$$

and for reservoir-type storage the assumption is made that

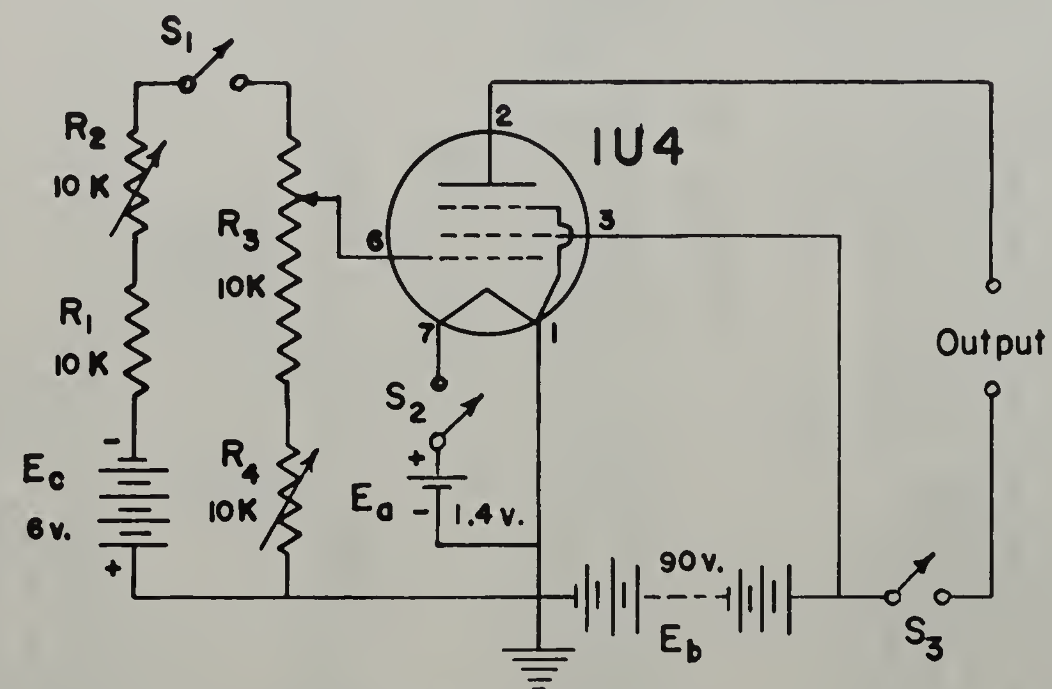
$$S = T_S O \tag{2}$$

where,

$I$ = inflow rate,
$O$ = outflow rate,
$S$ = storage volume, and
$T_S$ = a time constant.




(a) ROUTING CIRCUIT




(b) INFLOW CIRCUIT

**Parts List**

| | |
|---|---|
| $R_1$ | 10,000 ohm |
| $R_2$, $R_4$, $R_{12}$ | 10,000 ohm rheostat |
| $R_3$ | 10,000 ohm potentiometer |
| $R_5$, $R_7$ | 50 ohm |
| $R_6$, $R_8$ | 30 ohm rheostat |
| $R_9$ | 750 ohm rheostat |
| $R_{10}$ | 20,000 ohm rheostat |
| $R_{11}$ | 100,000 ohm rheostat |
| $R_{13}$ | 50,000 ohm rheostat |
| $C_1$, $C_2$, $C_3$, $C_4$ | 1000 $\mu$f, electrolytic, 50v. |
| $S_1$, $S_2$, $S_3$, $S_4$, $S_5$ | single pole, single throw |
| $S_6$, $S_7$, $S_8$, $S_9$ | single pole, double throw |

Note: All resistors 1/4 watt

Figure 6.--Circuit diagram of a test model analog (12).

The electric counterpart is achieved by placing a resistor (R) and capacitor (C) in parallel and introducing a current ($I_1$) into the network at the junction as shown in figure 7. From elementary electricity the following equations hold:

$$I_1 = I_2 + I_3$$

$$I_1 = I_3 + \frac{dQ}{dt}$$

$$V_r = V_c, \; V_r = I_3 R, \; V_c = \frac{Q}{C}$$

giving

$$Q = (RC)I_3 \tag{3}$$

where,

$R$ = resistance,
$C$ = capacitance,
$V_{r,c}$ = voltage across resistor or capacitor,
$Q$ = charge on capacitor, and
$I_{1,2,3}$ = current in branches.

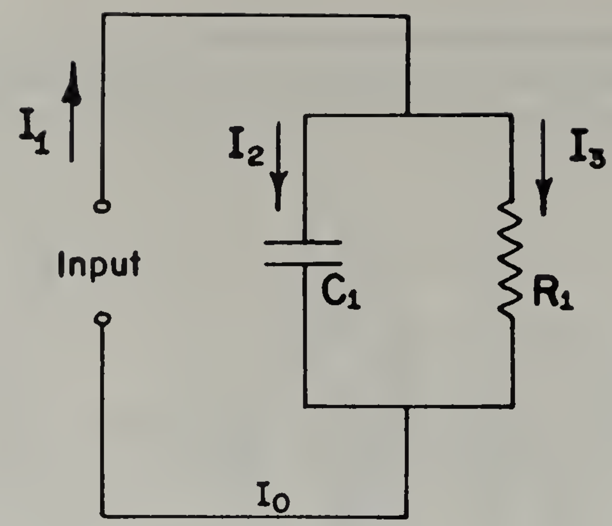
The comparison may now be made between the storage routing equations and the electric analogy:

| Storage routing | Electric circuit |
|---|---|
| $I = 0 + \frac{dS}{dt}$ | $I_1 = I_3 + \frac{dQ}{dt}$ |
| $S = T_s 0$ | $Q = (R\ C)\ I_3$ |

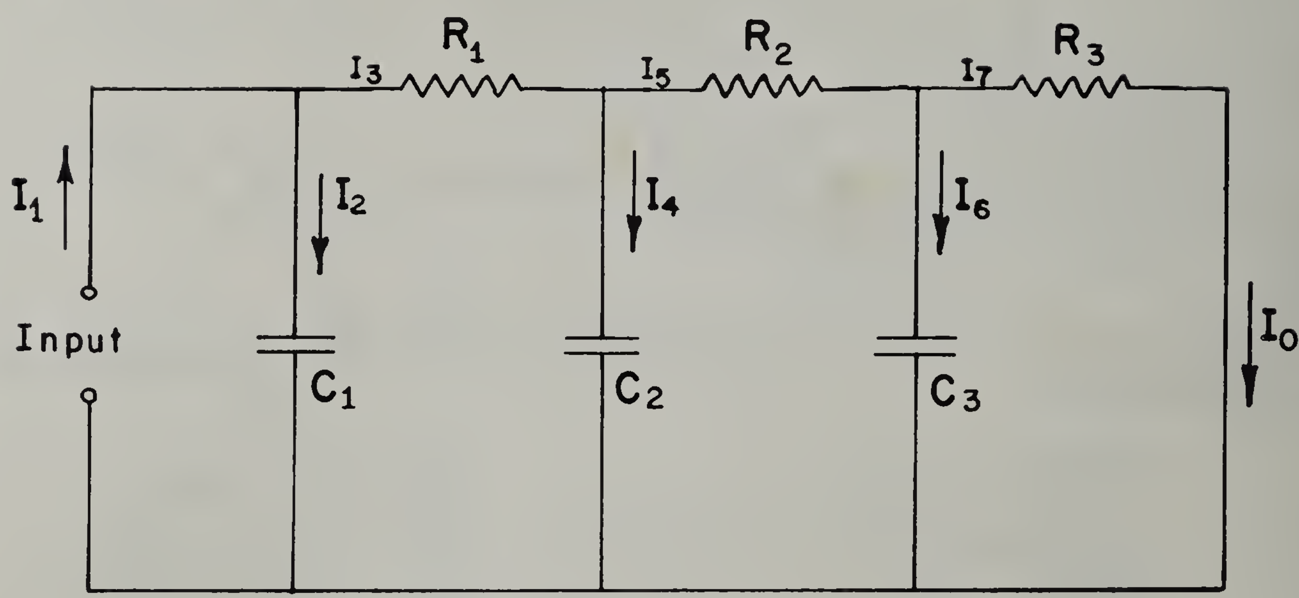
Therefore, the product of (RC) is equivalent to the time constant $T_s$ or $T_s = RC$. Watershed storage and river reaches can be considered as several successive reservoir-type stages of storage. This can be achieved analogously by connecting several (R C) networks so that $I_3$ for the first network becomes $I_1$ for the second, etc.

Briefly, the project involved building a similar but more complex electric analog than that developed by the Corps of Engineers at Portland (12) by means of the following modifications:

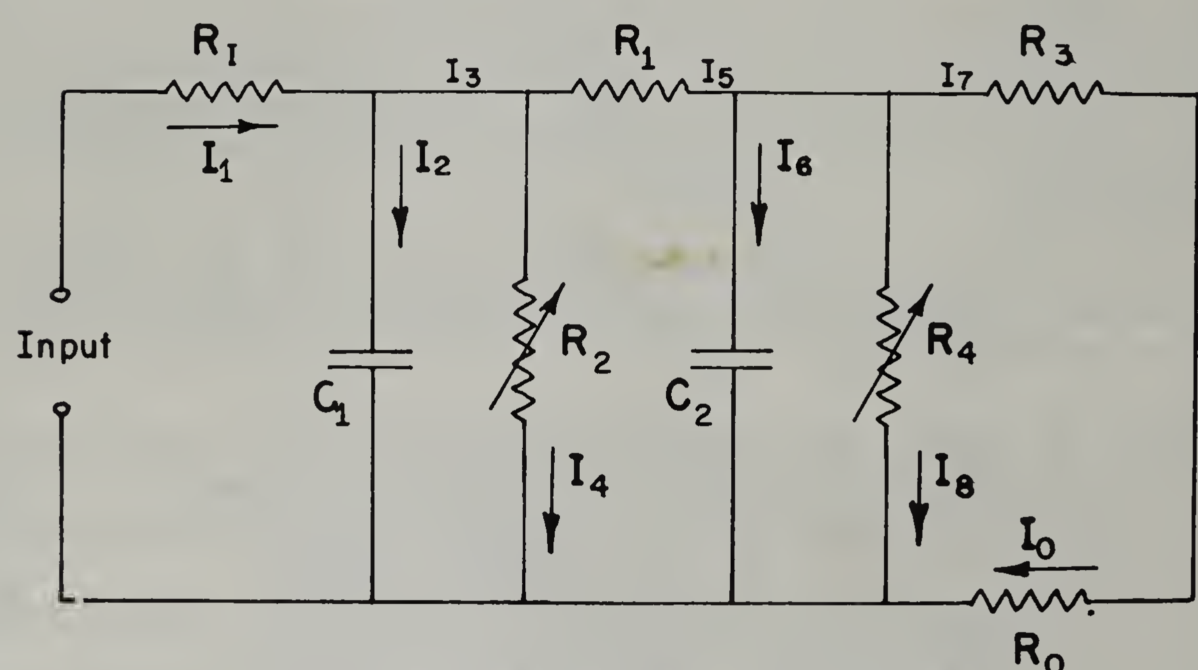
1. The number of stages was increased from two to five, and switches provided so that any number may be used from one to five.
2. The time constant is adjusted by a variable resistor with capacitors chosen to keep leakage to an insignificant level.
3. In addition to provision for expensive recorders to monitor inflow and outflow, two meters were added for direct reading.
4. Batteries were replaced by a power supply incorporated into the chassis.
5. An automatic input or "memory" unit was built to store 24 preset values.




(a) Single Stage




(b) Multiple Stages




(c) Variable Time of Storage

Figure 7.--Basic circuit diagrams.

A list of parts for the multiple-stage, storage-routing analog, as built by Beyers[5/] and reported by Chow (2) is shown below. Capacitors were selected so that they would operate linearly within their rating and duplicate the simple storage equation, equation 2, where $T_s$ = RC.

LIST OF PARTS FOR ANALOG MODEL

| | | |
|---|---|---|
| $V_1$ ---- | 6X4 vacuum tube | $ 1.65 |
| $V_2$ ---- | VR 105 gas regulator tube | 1.25 |
| $V_3$ ---- | 6BA6 vacuum tube | 2.00 |
| $T_1$ ---- | Pri.= 115 V, sec = 6.3V, 350 V.C.T., 50 ma. transformer | 7.35 |
| $M_1$ ---- | 0 - 1 ma. milliammeter | 10.40 |
| $M_2$ ---- | 0 - 1 ma. milliammeter | 10.40 |
| $S_1$ ---- | Dpdt switch | .40 |
| S ---- | Spdt switches | 1.00 |
| $L_1$ ---- | 48 m h inductance | 1.25 |
| $L_2$ ---- | 9.6 m h inductance | 1.00 |
| $C_1$,$C_5$ - | 200 $\mu$f, 250 WVDC capacitor | 8.00 |
| $C_2$ ---- | 300 $\mu$f, 200 WVDC capacitor | 2.80 |
| $C_3$,$C_4$,$C_6$,$C_7$ --- | 500 $\mu$f, 200 WVDC capacitor | 43.20 |
| $C_8$,$C_9$ --------- | 40 $\mu$f, 600 WVDC capacitor | 2.50 |
| $R_1$,$R_2$ --------- | 25 K ohm potentiometer | 2.50 |
| $R_3$,$R_4$,$R_5$ ------ | 10 K ohm potentiometer | 3.75 |
| $R_6$ ----------- | 5 K ohm potentiometer | 1.25 |
| $R_7$ ----------- | 100 K ohm potentiometer | 1.25 |
| $R_8$,$R_9$ --------- | 50 K ohm resistor | .30 |
| $R_{10}$ ---------- | 20 K ohm resistor | .15 |
| $R_{11}$ ---------- | 180 K ohm resistor | .15 |
| $R_{12}$ ---------- | 10 K ohm, 2-watt resistor | .85 |
| Total | | $103.40 |

Figure 8 shows photographs of the assembled analog model. In closeup it can be seen as desk-calculator size, 10 inches high by 18 inches long and 8 inches deep, weighing about 20 lb. On top is mounted a separate input device which feeds 24 preset values (on resistances) into the analog at 2.3-second intervals for automatic operation. The large knob on the front panel also serves the same purpose for hand operation where observations are noted on the panel-mounted milliammeters. Along the base of the front panel are the controls for setting various resistances and switches for connecting stages.

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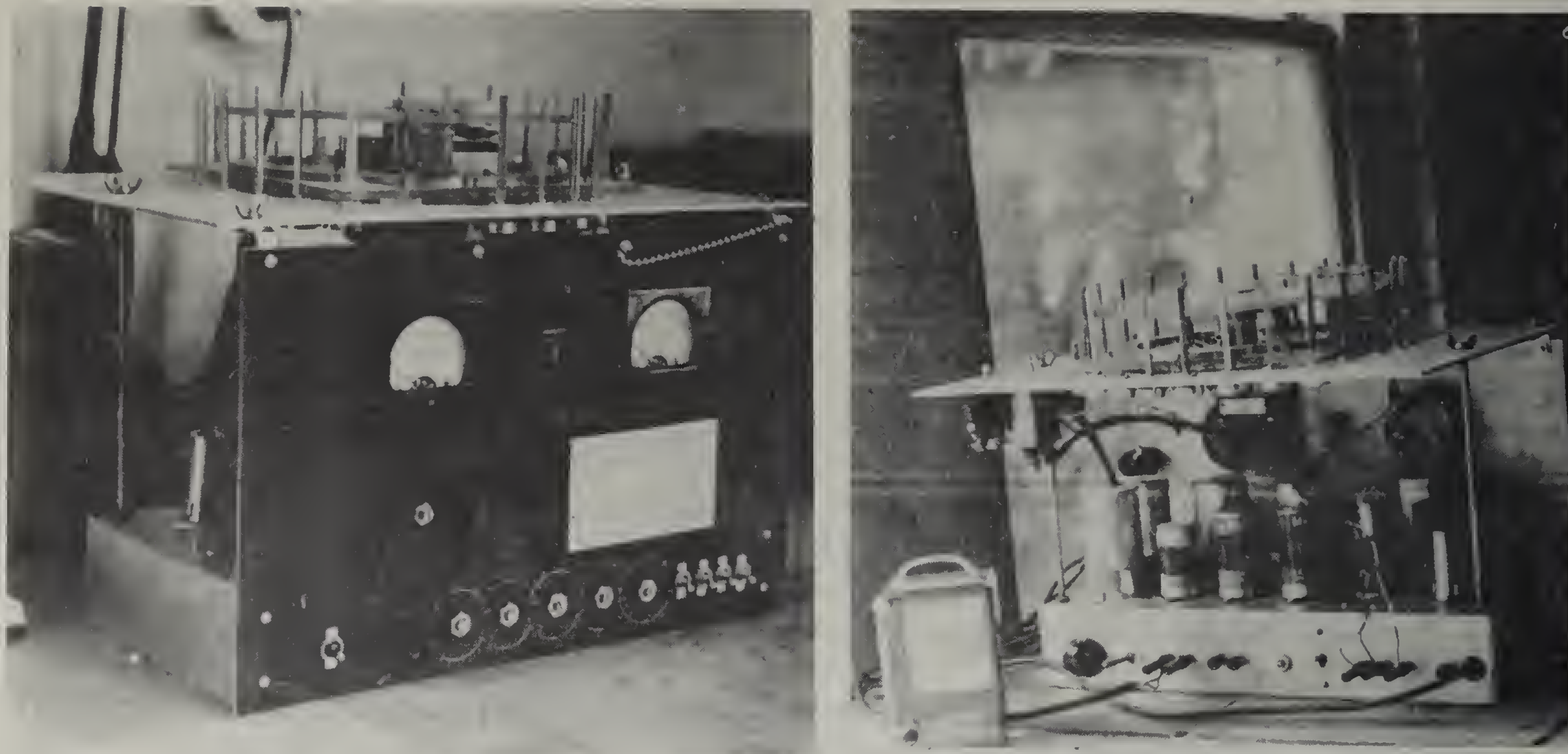
5/ Beyers, L. A. The electronic analogy to spring snow melt on a watershed. M.S. Thesis, University of Idaho, Moscow. [Unpublished.] 1962.



Front view　　　　　　　　　　Rear view

Figure 8.--Analog test model as viewed from front and rear.

The equipment design, as shown in figure 9, was based on a maximum inflow of 1 milliampere. This inflow figure was used to determine all ratings by inserting milliammeters in the inflow and outflow circuits. These provide continuous measurements from which data points can be read to reproduce the outflow hydrograph. In order to permit the operator to take a minimum of 20 data points for each run, an RC time of 10 seconds was chosen for each of the 5 stages desired.

The RC values of the components in each of the stages could be filled out naturally from the two choices already made. The ready accessibility of capacitors provided values of 200, 300, and 500 microfarads with a minimum working volts rating of 200. Since the capacitance per stage must increase in each succeeding stage and since leakage should be kept low, the section of series 200 $\mu$f. - 300 $\mu$f. - 500 $\mu$f. - 700 $\mu$f. - 1000 $\mu$f. was chosen. This selection forced the values of resistance for each stage. Consulting the schematic, figure 9, it can be found for RC = 10 sec. in each stage that:

(1) $C_1 = 200\text{x}10^{-6}$ farads, $R=R_1+R_2+R_3+R_4+R_5=50\text{x}10^3$ ohms;

(2) $C_2 = 300\text{x}10^{-6}$ farads, $R=R_2+R_3+R_4+R_5=33\text{x}10^3$ ohms;

(3) $C_3 = 500\text{x}10^{-6}$ farads, $R=R_3+R_4+R_5=20\text{x}10^3$ ohms;

(4) $C_4 = 700\text{x}10^{-6}$ farads, $R=R_4+R_5=14\text{x}10^3$ ohms; and

(5) $C_5 = 1000\text{x}10^{-6}$ farads, $R=R_5=10\text{x}10^3$ ohms.

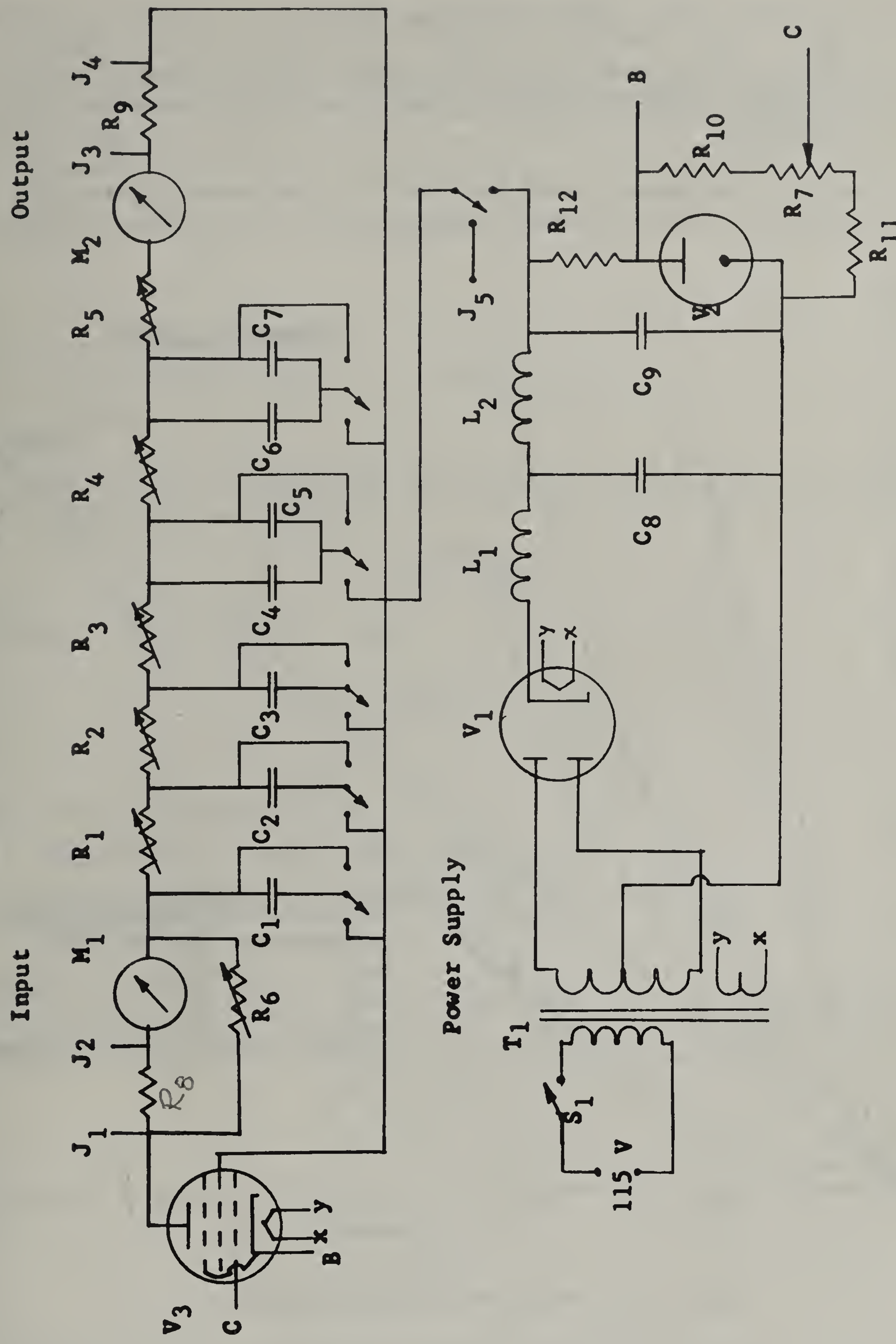



Figure 9.--Analog circuit, multiple-stage storage routing.

The plate current from a 6BA6 tube was chosen to provide an independently controlled inflow. This tube was designed for currents in this range and was selected because a pentode provides plate current independent of plate voltage. A VR105 tube permits a constant voltage source for the independent grid control of the plate current.

The design of this analog model was based on an overall precision of not less than 95 percent. The capacitor characteristics are shown in figure 10.

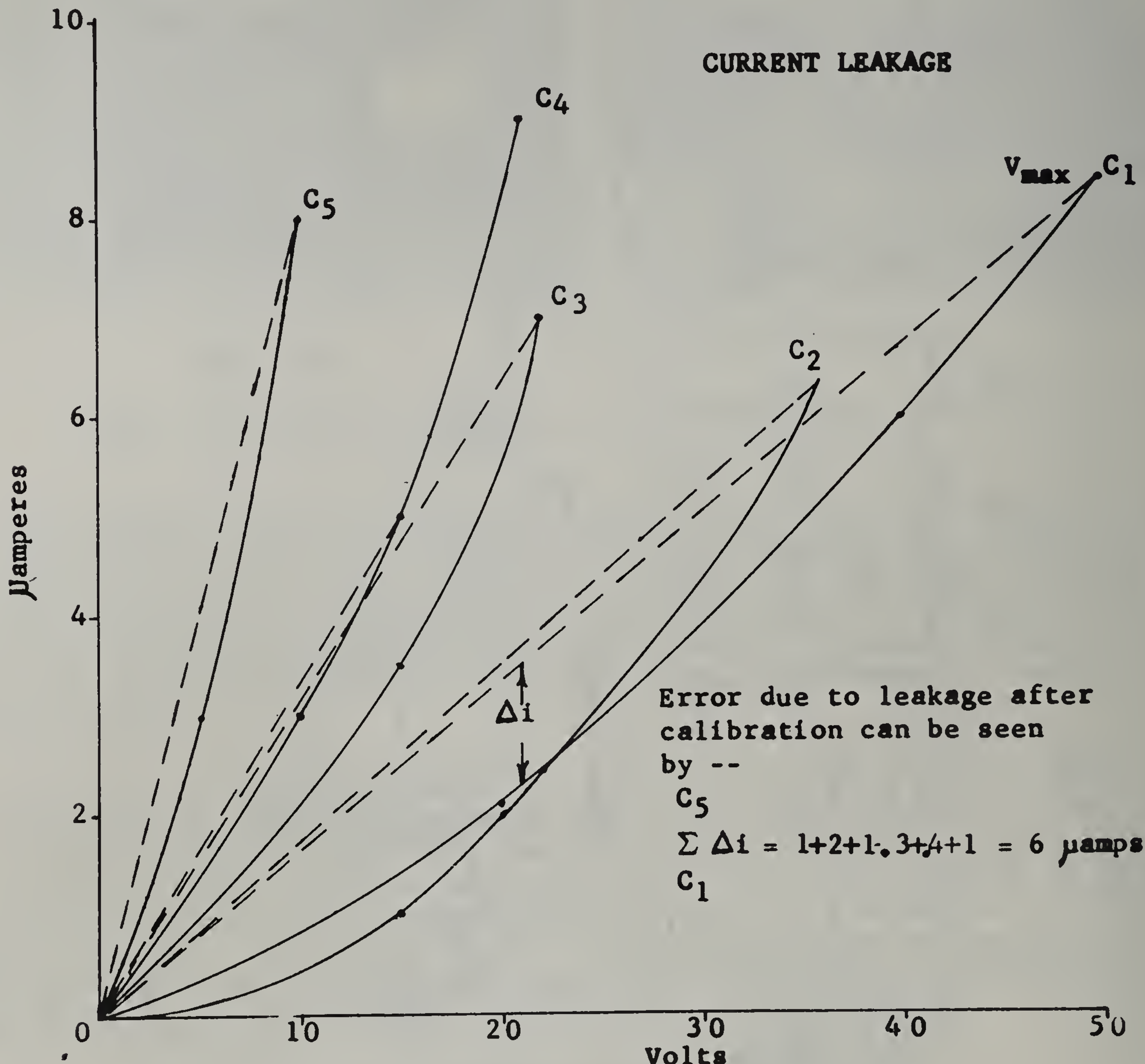



Figure 10.--Capacitor characteristics.

Leakage currents in the electrolytic capacitors introduce calibration and error considerations. The maximum voltages across the capacitors are: $C_1 = 50$ volts, $C_2 = 33$ volts, $C_3 = 20$ volts, $C_4 = 14$ volts, and $C_5 = 10$ volts. Leakage curves were plotted and approximated with straight lines for calibration. The maximum error through leakage is the sum of the deviations from the straight lines and is insignificant in this equipment.

In calibration tests of discharging storage, points observed along the recession curve were found to have a constant depletion ratio. The time of storage (storage divided by outflow) in seconds was linearly related to the product of capacitance times resistance or:

$$T_s = R\,C \tag{4}$$

where,

$R$ = ohms x $10^3$,
$C$ = farads x $10^{-6}$, and
$T_s$ = seconds.

Using this relation, the depletion of reservoir storage may be represented by the usual expression in the form

$$Q_t = Q_o e^{-t/RC} \tag{5}$$

or alternately in the general form

$$Q_t = Q_o K^t \tag{6}$$

where,

$Q_o$ = initial discharge,
$Q_t$ = discharge, (t) time units after $Q_o$,
$K$ = recession constant having a value less than one, and
$e$ = Napierian base.

From a typical depletion curve, figure 11, K can be computed by reading $Q_o$ and $Q_t$ separated by (t) units of time. Equations 4, 5, and 6 may be combined, assuming t = 1, to obtain the following equations:

$$K = e^{-1/T_s} \tag{7}$$

or

$$T_s = \frac{-1}{\ln K} \tag{8}$$

Calibration of the analog is for the purpose of verifying the linear operating characteristics of the selected capacitors. To have a constant time of storage, leakage of current across each capacitor should be at a minimum. The circuitry of the analog was designed so that voltages would not exceed the rating at which leakage becomes significant. Observations of the depletion of storage, for one or all stages of the completed RC-network, show that depletion ratios between 5-second current readings are essentially uniform (fig. 11).

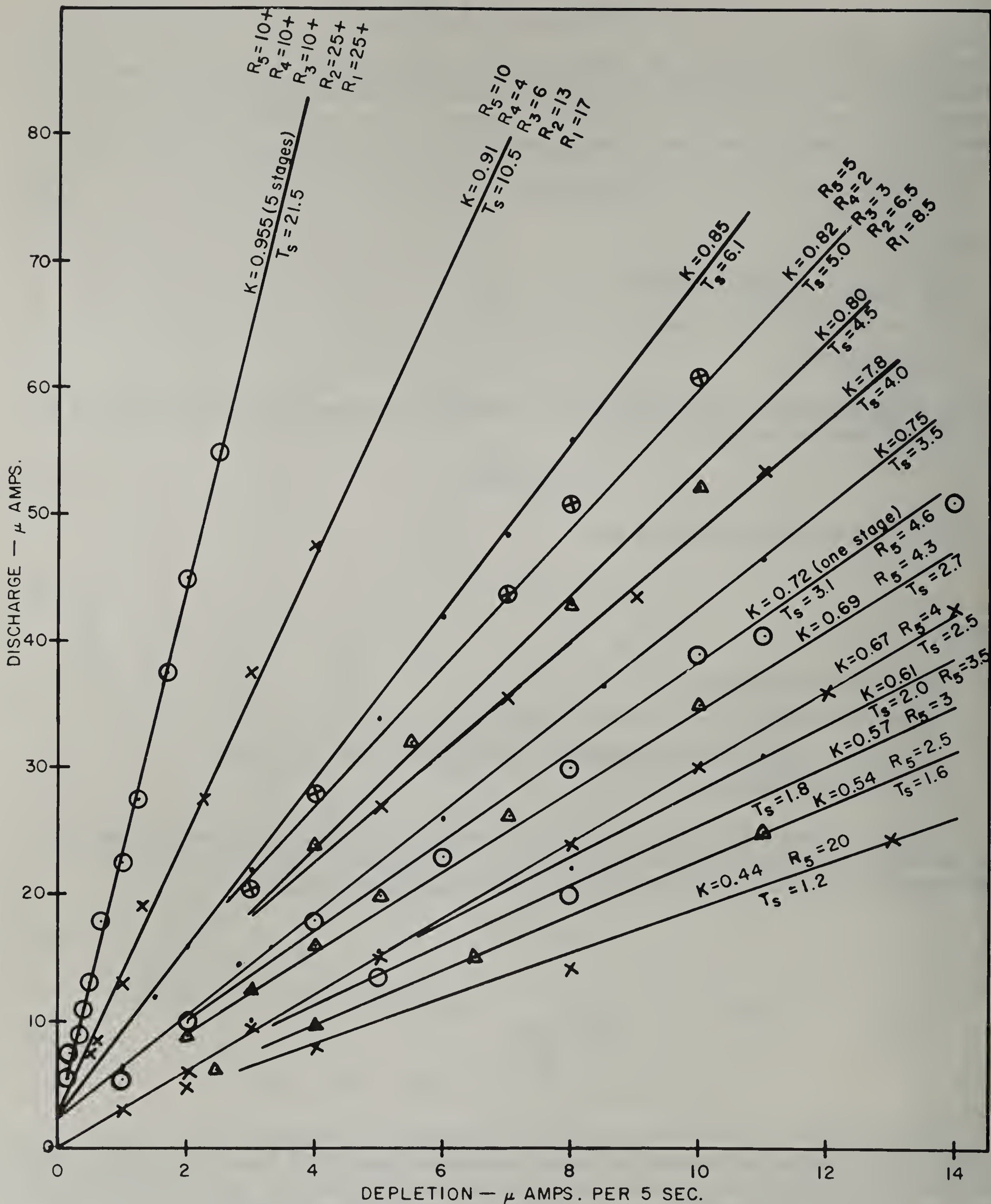



Figure 11.--Calibration of analog by storage depletion.

The time of storage is determined from values of capacitance multiplied by the variable resistance. Assuming that 5-second time intervals have been selected to represent 1-hour input, for example in one stage:

$$R_5 = 3.0,\ C_{1,5} = 2.7,\ RC = 8.1 \text{ second, equivalent } T_S = \frac{8.1}{5} = 1.6 \text{ hrs.}$$

## Input to Analog

An analog computer has been designed by Robinson and Beyers 6/ to calculate runoff increments from hourly rainfall rates (fig. 12) as input to the multiple-stage storage routing analog. The computer is based on the equation suggested by the Soil Conservation Service (16) and is stated as:

$$Q = \frac{P^2}{P + S} \tag{9}$$

where

Q = direct storm runoff,
P = storm rainfall, and
S = potential storage or maximum difference between P and Q.

The theory of the runoff analog computer is based on three elementary electrical relations: Ohm's Law and two formulas giving the equivalent resistance for series and parallel resistors. These three relations in equation form are as follows:

$$V = R\,I \text{ (Ohm's Law)} \tag{10}$$

$$R_{eq} = R_1 + R_2 + R_3 + \ldots \text{ (Resistors in series)} \tag{11}$$

$$R_{eq} = \frac{R_1\ R_2}{R_1+R_2} \text{ (Two resistors in parallel)} \tag{12}$$

Equation 9 may be written as

$$Q = P - \frac{S\ P}{P + S} \tag{13}$$

where the second term on the right in this equation is the same as equation 12, if S and P are interpreted as resistances. This form of the runoff equation was used to derive a formula for a special case of runoff increment, which may be easily generalized to apply to any increment. The derivation proceeded by letting $P_2 = p_1 + p_2$ and $P_3 = p_1 + p_2 + p_3$, where $p_1$, $p_2$, and $p_3$ are the average rainfall for the first three time periods of the rain following the initiation of runoff. Therefore, $P_2$ and $P_3$ represent the storm rainfall (accumulated rainfall) after the second and third time periods, respectively.

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6/ Robinson, A. D., and Beyers, L. A. An analog computer for calculating runoff increments from average rainfall rates. U.S. Soil and Water Cons. Res. Div. Unpublished Report, 10 pp. 1962. [Mimeographed.]

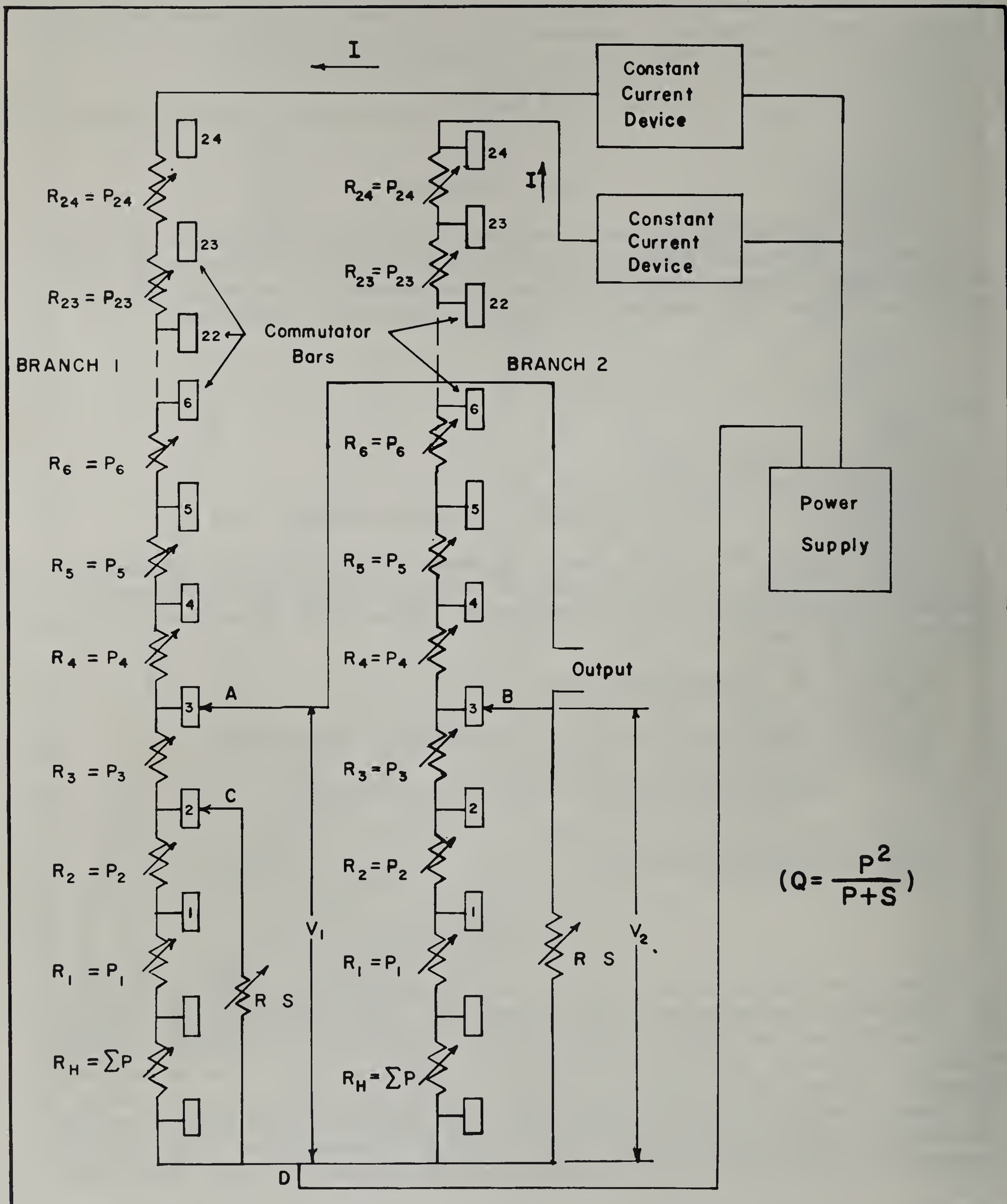


Figure 12.--Schematic diagram of computer: $Q = \frac{P^2}{P+S}$ .

The time periods used depend upon the desires of the computer operator or the available data. Using equation 13:

$$Q_2 = P_2 - \frac{S\ P_2}{P_2 + S} = p_1 + p_2 - \frac{S(p_1 + p_2)}{(p_1 + p_2) + S} \tag{14}$$

and

$$Q_3 = P_3 - \frac{S\ P_3}{P_3 + S} = p_1 + p_2 + p_3 - \frac{S\ (p_1 + p_2 + p_3)}{(p_1 + p_2 + p_3) + S} \tag{15}$$

where $Q_2$ and $Q_3$ are the total runoffs (accumulated runoff) after the second and third time periods, respectively. The runoff increment for the third time period is given by:

$$Q_3 - Q_2 = \left[p_1 + p_2 + p_3 - \frac{S(p_1 + p_2 + p_3)}{(p_1 + p_2 + p_3) + S}\right] - \left[p_1 + p_2 - \frac{S(p_1 + p_2)}{(p_1 + p_2) + S}\right] \tag{16}$$

which may be reduced to:

$$Q_3 - Q_2 = p_3 - \frac{S(p_1 + p_2 + p_3)}{(p_1 + p_2 + p_3) + S} + \frac{S(p_1 + p_2)}{(p_1 + p_2) + S} \tag{17}$$

In developing an electrical analogy to equation 17, the assumption was made that the currents labeled I in Branch 1 and Branch 2 of figure 12 are equal, constant, and independent of any changes of resistance, power supply voltages, or brush positions. The reason for the constant current devices is to insure this. Another assumption was that both resistances labeled $R_H = \Sigma p$ are zero. Referring to the circuit in figure 12 and using equations 11 and 12, the equivalent resistance between points A and D is found to be

$$R_{eq_{AD}} = R_3 + \frac{(R_1 + R_2)\ R}{(R_1 + R_2) + R} \tag{18}$$

and between points B and D to be

$$R_{eq_{BD}} = \frac{(R_1 + R_2 + R_3)\ R}{(R_1 + R_2 + R_3) + R} \tag{19}$$

Applying equations 10 and 18, the potential difference between points A and D is found to be

$$V_1 = I\left[R_3 + \frac{(R_1 + R_2)\ R}{(R_1 + R_2) + R}\right]$$

and between points B and D to be

$$V_2 = I\left[\frac{(R_1 + R_2 + R_3)\ R}{(R_1 + R_2 + R_3) + R}\right]$$

As can be seen from equations 10 and 19, the potential difference between points A and B, which is one of the output voltages of the computer, is

$$V_1 - V_2 = I\left[R_3 + \frac{(R_1 + R_2)\,R}{(R_1 + R_2) + R}\right] - I\left[\frac{(R_1 + R_2 + R_3)\,R}{(R_1 + R_2 + R_3) + R}\right] \tag{20}$$

This can be written as

$$V_1 - V_2 = I\left[R_3 + \frac{(R_1 + R_2)\,R}{(R_1 + R_2) + R} - \frac{(R_1 + R_2 + R_3)\,R}{(R_1 + R_2 + R_3) + R}\right] \tag{21}$$

If the pairs of resistors labeled $R_1$, $R_2$, and $R_3$, and R are chosen such that $R_1 = p_1$, $R_2 = p_2$, $R_3 = p_3$, and $R = S$, equation 21 may be written as

$$V_1 - V_2 = I\left[p_3 + \frac{(p_1 + p_2)\,S}{(p_1 + p_2) + S} - \frac{(p_1 + p_2 + p_3)\,S}{(p_1 + p_2 + p_3) + S}\right] \tag{22}$$

An examination of equations 17 and 22 shows that $V_1 - V_2$ is proportional to $Q_3 - Q_2$. This fact is the basis for the design of the computer.

The above derivation is for the brush positions shown in figure 12. If, for example, the brushes labeled C and A are on commutator segments 5 and 6, respectively, in Branch 1, and if at the same time the brush labeled B is on commutator segment 6 in Branch 2, then the output voltage is proportional to $Q_6-Q_5$. Sweeping the brushes at a uniform rate will, therefore, produce a voltage output which is a step function. The height of the individual step is the runoff increment, and the width of the step is the time unit. This is the same as the time unit over which the inputs are averaged.

In deriving equation 22 it was assumed that the resistances labeled $R_H = \Sigma p$ in figure 12 were zero. This would be the case if the analog were being programmed to start at the beginning of a rainfall. When the computer is programmed to start at a time other than the beginning of the rainfall, the two resistances $R_H$ are set equal to the accumulated P at this time. This feature also extends the total rainfall time for which the computer can be used.

## Operation of Analog

Operation of the analog model is very simple for reproducing recorded hydrographs from small watersheds. Given the time distribution of rainfall for routing of excess rainfall into a downstream hydrograph, output can be computed from input by the proper choice of only three values:

1. The <u>potential storage</u> for solution of equation 9.

2. The _time_ _of_ _storage_, $T_s$, to determine the RC value so that resistances can be set on the analog.

3. The _number_ _of_ _stages_ or the lag time which may be estimated from the length of channels, slope, area, etc.

By plugging a recorder in the output jacks of the analog, the computed hydrograph is quickly obtained and after a few trial-and-error solutions the best choice of arbitrary values is soon indicated. Only one choice of RC values, number of stages, and potential storage will best duplicate the available hydrographs in general shape and time of occurrence. These values can be said to have scaled the analog so that it is now a model of that particular watershed. Thus, any other rainfall record could be converted into a hydrograph from that watershed or any other ungaged drainage with similar soil, cover, geology, drainage density, slope, area, etc.

## EXAMPLES OF HYDROGRAPHS

The practical application of this analog model can be illustrated by a sample computation (table 1) and by typical hydrographs from the South Fork of the Palouse River from Moscow, Idaho, to Pullman, Wash., as shown by figures 13 and 14. Hourly rainfall values from recording gages on the watershed were first tabulated, then converted to runoff (i.e. excess precipitation) by use of equation 9, in the form of $\Sigma Q = \Sigma P + S$, after estimating the potential storage (S). Finally, the increments of runoff ($\Delta Q$) were entered into the analog after conversion to milliamperes and routed through a number of stages and time of storage so that the final downstream generated hydrograph agrees with the actual. For reproduction of hydrographs on tributaries of the Palouse River, a value of $T_s$ ranging from 5 to 10 hours and the use of 5 to 10 stages are required for drainage areas of 34 to 84 square miles. The physical significance of the time of storage, $T_s$, may be characteristic of land form, soil mantle, and cover type. The number of stages indicates the length of channel--that is, the number of routing reaches.

## CONCLUSIONS

The application of electric analog methods to the solution of a complex network of channels and reservoirs is relatively new in the United States. Because the future of these devices seems promising for both ground water and surface routing, a test model of multiple stages of linear reservoirs was assembled simply and inexpensively. Recommendations resulting from testing this model analog are:

1. The number of stages should be increased to 10 or more.

2. Leakage currents could be decreased by greater capacitance or by more expensive capacitors.

3. The time of storage should be varied exponentially.

TABLE 1.--Analog hydrograph computations on Palouse River, Fourmile Creek at Shawnee, Wash., April 20, 1937

| Time t | Watershed | | | Analog | | Hydrograph |
|---|---|---|---|---|---|---|
| | Rain | Runoff[1] | | Input[2] | Output[3] | Computed[4] |
| | $\Sigma P$ | $\Sigma Q$ | $\Delta Q_1$ | $\Delta Q_1 \times 50$ | $\Delta Q_2$ | $\Delta Q_2 \div 50$ |
| | In. | In. | In. | Ma. | Ma. | In./hr. |
| | | | 0.00002 | 0.00 | 0.00 | 0.0000 |
| 1000 | 0.01 | 0.00002 | | | | |
| | | | .00017 | .01 | .00 | .0000 |
| 1100 | .03 | .00019 | | | | |
| | | | .00034 | .02 | .00 | .0000 |
| 1200 | .05 | .00053 | | | | |
| | | | .00082 | .04 | .00 | .0000 |
| 1300 | .08 | .00135 | | | | |
| | | | .00165 | .08 | .00 | .0005 |
| 1400 | .12 | .0030 | | | | |
| | | | .0030 | .15 | .01 | .0007 |
| 1500 | .17 | .0060 | | | | |
| | | | .0022 | .11 | .02 | .0009 |
| 1600 | .20 | .0082 | | | | |
| | | | .0066 | .33 | .03 | .0011 |
| 1700 | .27 | .0148 | | | | |
| | | | .0166 | .83 | .04 | .0013 |
| 1800 | .40 | .0314 | | | | |
| | | | .0149 | .79 | .07 | .0019 |
| 1900 | .49 | .0463 | | | | |
| | | | .0037 | .19 | .11 | .0026 |
| 2000 | .51 | .0500 | | | | |
| | | | .0020 | .10 | .18 | .0041 |
| 2100 | .52 | .0520 | | | | |
| | | | -- | -- | .24 | .0053 |
| 2200 | -- | -- | | | | |
| | | | -- | -- | .27 | .0059 |
| 2300 | -- | -- | | | | |
| | | | -- | -- | .27 | .0059 |
| 2400 | -- | -- | | | | |
| | | | -- | -- | .24 | .0053 |

1/ $\Sigma Q = \frac{\Sigma P^2}{\Sigma P + 4.68}$

2/ Factor for full-scale deflection of one milli-amp. = 50.

3/ 10 stages at 2.5 seconds = 25 seconds, RC = 2.5 seconds, $T_s$ = 5 hours.

4/ $\Delta Q_2/50$ + 0.0005 in./hour.

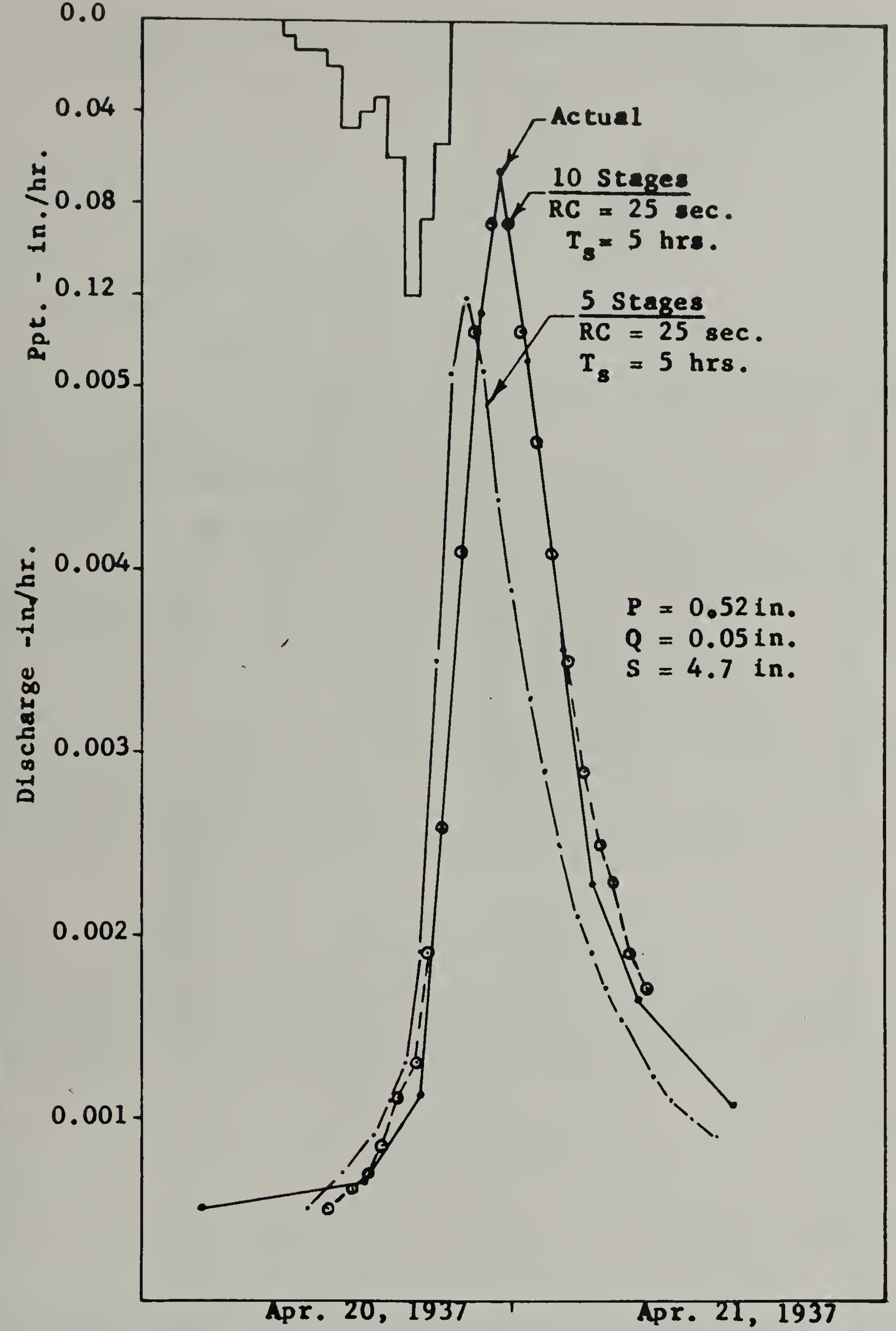


Figure 13.--Hydrographs: Fourmile Creek at Shawnee, Wash. (71.6 sq. mi.)

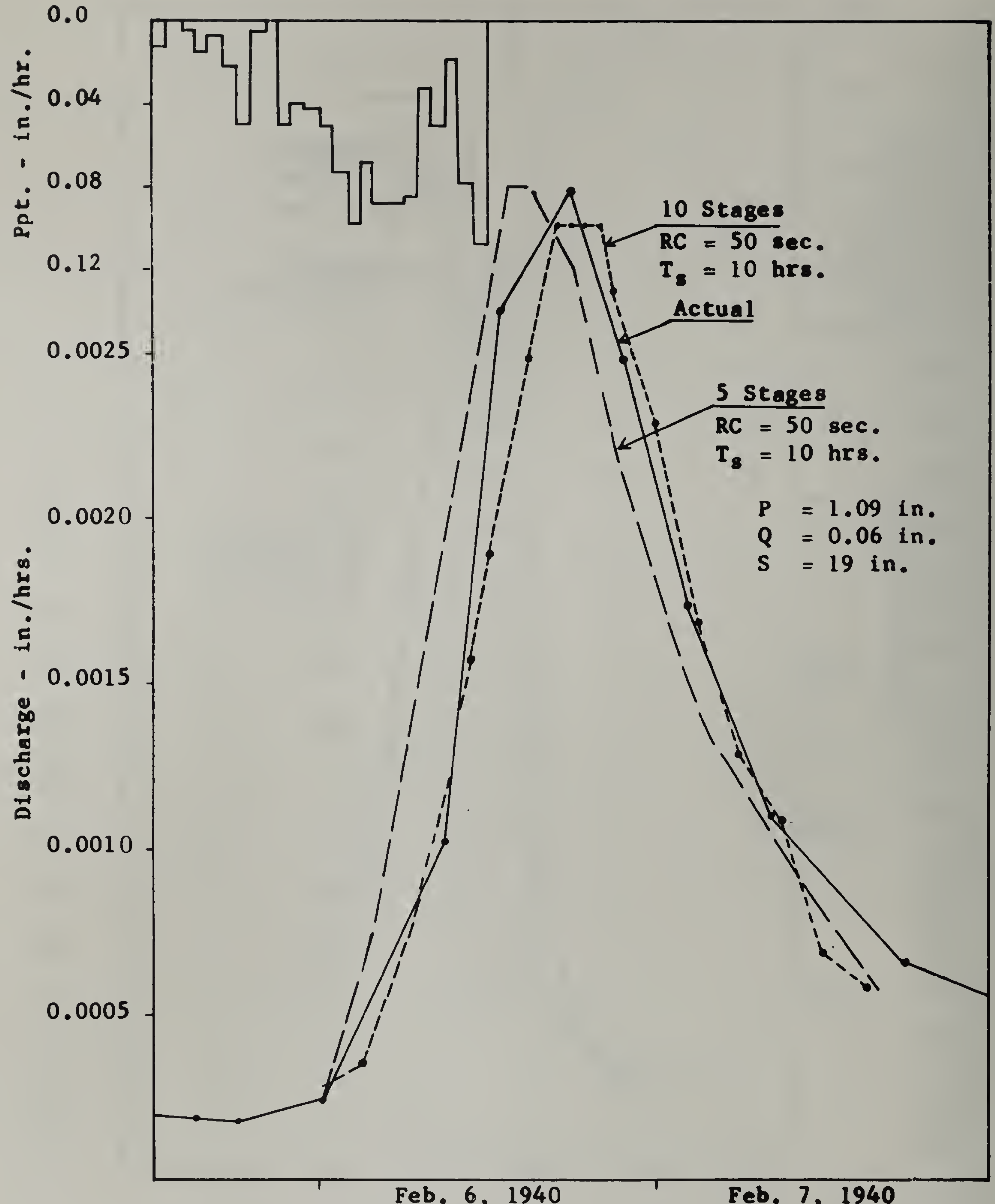


Figure 14.--Hydrographs: South Fork Palouse River at Pullman, Wash. (84.4 sq. mi.)

The analog should be thought of, not as a mathematical computer, but as a "tool to think with" in terms of the physical system. The operation of the analog model is not based on man-devised algebra, as is that of the digital computer, but is governed by the physical laws of the electrical circuits.

An outstanding advantage of the analog model is that it can be constructed of standard radio parts and can be operated by personnel with only routine engineering training. The use of the analog model is a never-ending game; there must always be feedback between the user and the system.

Some advantages of this multiple-stage, storage-routing analog are:

1. Flexible operation to meet variable storage-discharge relations.
2. Application to all routing where analytical or graphical solutions were previously required.
3. Success in routing through channels or basin storage for rain or snowmelt.
4. A saving in operating time by replacing tedious graphical methods, thus making possible many trial routings that otherwise would not be attempted.
5. Simplicity and cost of about $100 for parts.

An improved analog model would be particularly useful for:

1. Calibrating experimental watersheds before new rainfall or runoff data are available.
2. Forecasting water supplies or estimating design floods.
3. Demonstrating routing procedures to local groups.

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